

Can the Quantum Measurement Problem be resolved within the framework of Schroedinger Dynamics and Quantum Probability?

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Abstract We provide an affirmative answer to the question posed in the title. Our argument is based on a treatment of the Schroedinger dynamics of the composite, S_c , of a quantum microsystem, S , and a macroscopic measuring apparatus, \mathcal{I} , consisting of N interacting particles. The pointer positions of this apparatus are represented by subspaces of its representative Hilbert space that are simultaneous eigenspaces of coarse-grained intercommuting macroscopic observables. By taking explicit account of their macroscopicity, we prove that, for suitably designed apparatus \mathcal{I} , the evolution of the composite S_c leads both to the reduction of the wave-packet of S and to a one-to-one correspondence between the resultant state of this microsystem and the pointer position of \mathcal{I} , up to utterly negligible corrections that decrease exponentially with N .

Keywords: Schroedinger dynamics of microsystem-cum-measuring instrument, macroscopic phase cells as pointer positions, macroscopic decoherence, reduction of wave packet of microsystem, large deviations.

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1. Introductory Discussion

According to Von Neumann's [1] phenomenological picture of the measurement process, the coupling of a microsystem, S , to a measuring instrument, \mathcal{I} , leads to the following two effects.

(I) It converts a pure state of S , as given by a linear combination $\sum_{r=1}^n c_r u_r$ of its orthonormal energy eigenstates u_r , into a statistical mixture of these states for which $|c_r|^2$ is the probability of finding this system in the state u_r . This effect is often termed the 'reduction of the wave packet'.

(II) It sends a certain set of classical, i.e. intercommuting, macroscopic variables M of \mathcal{I} to values, indicated by pointers, that specify which of the states u_r of S is realised.

This leaves us with the following basic question of quantum measurement theory, which is clearly pertinent to that of the completeness of quantum mechanics.

Q. Can the standard quantum dynamics of the composite $(S+\mathcal{I})$, allied to a suitable choice of the macroscopic observables M , lead to the effects (I) and (II)?

This is the contentious question that we address here and, as in [2,3], our response to it is affirmative, as is that of some other authors [4-6]. Our present objective is to describe the mathematical structure of our general argument that leads to this conclusion and that is supported by treatments of some concrete models. We remark that our affirmative response to *Q* conflicts with that of Von Neumann [1] and Wigner [7], who argued that the observation of the pointer of \mathcal{I} requires another measuring instrument, \mathcal{I}_2 , which in turn requires yet another instrument, and so on, in such a way that the whole process involves an infinite regression ending up in the observer's brain!* In our view, the essential flaw in the Von Neumann- Wigner theory is its failure to take explicit account of the macroscopicity of the observables corresponding to the pointer positions of the measuring apparatus. By contrast, we build this macroscopicity into our treatment, and thereby evince a qualitative difference between the characteristic properties of the microsystem S and the macrosystem \mathcal{I} that removes the need for a Von Neumann-Wigner regression.

As a preliminary to setting up a mathematical theory of the measurement process, we note that it is clear from the works of Bohr [13], Jauch [14] and Van Kampen [5] that such a theory demands both a characterisation of the macroscopicity of the observables M and an amplification property of the $S-\mathcal{I}$ coupling whereby different microstates of S give rise to macroscopically different states of \mathcal{I} . Evidently, this implies that the initial state in which \mathcal{I} is prepared must be unstable against microscopic changes in the state of S . On the other hand, as emphasised by Whitten- Wolfe and Emch [15, 16], the correspondence between the microstate of S and the eventual observed macrostate of \mathcal{I} must be stable against macroscopically small changes in the initial state of this instrument, of the kind that are inevitable in experimental procedures. Thus, the system $(S + \mathcal{I})$ must exhibit a striking combination of stability and instability properties.

Rigorous constructive treatments of the measurement process, which take account of the above considerations, have been provided by Hepp [17] and Whitten-Wolfe and Emch [15, 16] on the basis of models for which \mathcal{I} is idealised as an infinitely extended system of finite density, for which the macroscopic observables M are intercommuting global intensive ones. This picture of \mathcal{I} corresponds to that employed for the statistical mechanical description of large systems in the thermodynamic limit [18-20], and it has the merit of sharply distinguishing between macroscopically different states, since different values of M correspond to disjoint primary representations of the observables. Moreover, in the treatments of the measurement problem based on this picture, the models of Hepp [17] and Whitten-Wolfe and Emch [15, 16] do indeed exhibit the required reduction of the wave-packet and the one-to-one correspondence between the pointer position of \mathcal{I} and the resultant state of S ; and these results are stable against all localised perturbations of the initial state of \mathcal{I} . On the debit side, however, Hepp's model requires an infinite time for

* Others have taken the view that the measurement problem cannot be resolved without modification of the Schroedinger dynamics of S_c , due to *either* its interaction with the 'rest of the Universe' [8-11] *or* a certain postulated nonlinearity that leads to a classical deterministic evolution of its macroscopic observables [12].

the measurement to be effected (cf. Bell [21]), while although that of Whitten-Wolfe and Emch achieves its measurements in finite times, it does so only by dint of a physically unnatural, globally extended $S - \mathcal{I}$ interaction.

In view of these observations, we base our treatment of the measuring process on the generic model of $S_c = (S + \mathcal{I})$ for which \mathcal{I} is a large but finite N -particle system. Our main aim is to determine whether there are viable conditions under which this model yields the same essential results as the infinite one, but within a finite realistic observational time. In fact, we have achieved this aim in recent works [2, 3], in which we showed that the quantum mechanics of the finite model does indeed contain the structures required for an affirmative answer to the question \mathcal{Q} , as illustrated by an explicit treatment of the finite version of the Coleman-Hepp model [17]. This result provides rigorous mathematical substantiation of the arguments of Refs. [4-6] which led to the same main conclusion. A key feature of our treatment is the representation of the macro-observables M and the pointer positions of \mathcal{I} within the framework proposed by Van Kampen [22] and Emch [23], whereby M comprises a set of coarse-grained intercommuting extensive observables and the pointer positions of \mathcal{I} correspond to their simultaneous eigenspaces. These are mutually orthogonal subspaces of the Hilbert space of the pure states of \mathcal{I} and are the natural analogues of classical phase cells. Most importantly, their dimensionalities are astronomically large, increasing exponentially with N , and their role in the present treatment is analogous to that of the inequivalent representation spaces in the infinite system models of Refs. [15- 17]. As a result, the finite system model yields the essential positive results of the infinite one, but within a finite, realistic observational time. To be precise, it exhibits the above properties (I) and (II), in a stable manner, up to corrections that decrease exponentially with N and that are therefore utterly negligible by any standards of experimental physics.

The present note is devoted to a description of our mathematical scheme that has led to these results. We start in Section 2 by constructing the generic model of S_c and formulating both the time-dependent expectation values of the observables of S and their conditional expectation values, given the values of the macro-observables M of \mathcal{I} , subject to the assumption that S and \mathcal{I} are independently prepared and then coupled together at time $t = 0$. In Section 3, we formulate the conditions on the dynamics of the model and the structure of the macro-observables M under which it exhibits the properties (I) and (II) of a measurement process in a stable manner. We then briefly describe two models that fulfill these conditions, referring the reader to other articles for their detailed treatments. In Section 4, we probe further into the probabilistic structure of the model in order to pin-point, in general terms, the source of the effects (I) and (II). There we find that this is provided by a *large deviation* principle, which represents a rather general collective property of many-particle systems [24] and constitutes a generalisation to certain nonequilibrium states of Einstein's formula, $P = \text{const.} \exp(S)$, which relates the equilibrium probability distribution, P , of chosen macroscopic variables to the entropy function, S , of these variables. We conclude, in Section 5, with a brief resume of the picture presented here.

2. The Generic Model

We assume that the algebras of bounded observables of the microsystem S , the instru-

ment \mathcal{I} and their composite $S_c = (S + \mathcal{I})$ are those of the bounded operators in separable Hilbert spaces \mathcal{H}, \mathcal{K} and $\mathcal{H} \otimes \mathcal{K}$, respectively. Correspondingly, the states of these systems are represented by the density matrices in the respective spaces. The density matrices for the pure states of any of these systems are then the projection operators $P(f)$ of their normalised vectors f . For simplicity we assume that \mathcal{H} is of finite dimensionality n .

We base the macroscopic description of \mathcal{I} pertinent to the measuring process on an abelian subalgebra \mathcal{M} of \mathcal{B} , which is generated by coarse-grained macroscopic observables (cf. [22, 23]): these are typically extensive variables of parts or the whole of \mathcal{I} . The choice of \mathcal{M} yields a partition of \mathcal{K} into the simultaneous eigenspaces \mathcal{K}_α of its elements. These subspaces of \mathcal{K} are termed ‘phase cells’ as they are the canonical analogues of classical phase cells. We take them to represent the macrostates of \mathcal{I} and we assume that they are unequivocally indicated by the ‘pointer positions’ of this instrument. Most importantly, the dimensionality of each \mathcal{K}_α is astronomically large, since it increases exponentially with the corresponding entropy and thus with N .

Since \mathcal{I} is designed so that the pointer readings are in one-to-one correspondence with the eigenstates u_1, \dots, u_n of S , we assume that the index α of its macrostates also runs from 1 to n . Hence, denoting the projection operator for \mathcal{K}_α by Π_α , it follows from the above specifications that

$$\Pi_\alpha \Pi_\beta = \Pi_\alpha \delta_{\alpha\beta}, \quad (1)$$

$$\sum_{\alpha=1}^n \Pi_\alpha = I_{\mathcal{K}} \quad (2)$$

and that each element M of \mathcal{M} takes the form

$$M = \sum_{\alpha=1}^n M_\alpha \Pi_\alpha, \quad (3)$$

where the M_α ’s are scalars.

We assume that S_c is a conservative system, whose Hamiltonian operator H_c , in $\mathcal{H} \otimes \mathcal{K}$, takes the form

$$H_c = H \otimes I_{\mathcal{K}} + I_{\mathcal{H}} \otimes K + V, \quad (4)$$

where H and K are the Hamiltonians of S and \mathcal{I} , respectively, and V is the S – \mathcal{I} interaction energy. Further, we assume that \mathcal{I} is an instrument of the first kind [14], in that the interaction V induces no transitions between the eigenstates of H . Thus, since the latter comprise an orthogonal basis set (u_1, \dots, u_n) of \mathcal{H} , with energy levels $(\epsilon_1, \dots, \epsilon_n)$, respectively, the operators H and V take the forms $\sum_{r=1}^n \epsilon_r P(u_r)$ and $\sum_{r=1}^n P(u_r) \otimes V_r$, respectively, where the V_r ’s are self-adjoint operators in \mathcal{K} . Hence, by Eq. (4), H_c reduces to the form $H_c = \sum_{r=1}^n P(u_r) \otimes K_r$, where

$$K_r = K + V_r + \epsilon_r I_{\mathcal{K}}. \quad (5)$$

Consequently the one-parameter unitary group $\{U_c(t) = \exp(iH_c t) | t \in \mathbf{R}\}$, which governs the dynamics of S_c , is given by the formula

$$U_c(t) = \sum_{r=1}^n P(u_r) \otimes U_r(t), \quad (6)$$

where

$$U_r(t) = \exp(iK_r t). \quad (7)$$

Note. Eqs (5)-(7) signify that different eigenstates u_r of S give rise to different evolutions of \mathcal{I} . This is crucial to the realisability of the effect (II) of Sec. 1, whereby the pointer of \mathcal{I} is driven into a position determined by the eigenstate of S .

Initial Conditions. We assume that the the systems S and \mathcal{I} are coupled together at time $t = 0$ following independent preparation of S in a pure state and \mathcal{I} in a mixed one, as represented by a normalised vector ψ and a density matrix Ω , respectively. Thus the initial state of the composite S_c is given by the density matrix

$$\Phi = P(\psi) \otimes \Omega \quad (8)$$

and its evolve at time t (> 0) is

$$U_c^*(t) \Phi U_c(t) := \Phi(t). \quad (9)$$

Further, since ψ is a normalised vector, it is a linear combination of the basis vectors (u_1, \dots, u_n) and hence takes the form

$$\psi = \sum_{r=1}^n c_r u_r, \quad (10)$$

where $\sum_{r=1}^n |c_r|^2 = 1$. Hence, by Eqs. (8)-(10),

$$\Phi(t) = \sum_{r,s=1}^n \bar{c}_r c_s P_{r,s} \otimes \Omega_{r,s}(t), \quad (11)$$

where $P_{r,s}$ is the operator in \mathcal{H} defined by the equation

$$P_{r,s} f = (u_s, f) u_r \quad \forall f \in \mathcal{H} \quad (12)$$

and

$$\Omega_{r,s}(t) = U_r^*(t) \Omega U_s(t). \quad (13)$$

We note that $\Omega_{r,r}(t)$ is just the evolve of Ω corresponding to the eigenstate u_r of S .

Expectation and Conditional Expectation Values of Observables. The observables of S_c with which we shall be concerned are just the self-adjoint elements of $\mathcal{A} \otimes \mathcal{M}$. Their expectation values for the time-dependent state $\Phi(t)$ are given by the formula

$$E(A \otimes M) = \text{Tr}(\Phi(t)[A \otimes M]) \quad \forall A \in \mathcal{A}, M \in \mathcal{M}. \quad (14)$$

In particular, the expectation values of the observables of S are given by the equation

$$E(A) = E(A \otimes I_{\mathcal{K}}) \quad \forall A \in \mathcal{A}, \quad (15)$$

while the probability that the macrostate of \mathcal{I} corresponds to the cell \mathcal{K}_α is

$$w_\alpha = E(I_{\mathcal{H}} \otimes \Pi_\alpha). \quad (16)$$

Further, in view of the abelian property of the algebra \mathcal{M} , the functional E induces a conditional expectation of $A(\in \mathcal{A})$ with respect to \mathcal{M} , namely a linear map, $E(\cdot|\mathcal{M})$, of \mathcal{A} into \mathcal{M} that preserves positivity and normalisation and that satisfies the compatibility condition

$$E(E(A|\mathcal{M})M) = E(A \otimes M) \quad \forall A \in \mathcal{A}, M \in \mathcal{M}. \quad (17)$$

Moreover, since $E(A|\mathcal{M})$ is an element of \mathcal{M} and therefore takes the form $\sum_{\alpha=1}^n f_\alpha(A) \Pi_\alpha$, where the f_α 's are linear, positive, normalised functionals on \mathcal{A} , it follows from this observation and Eqs. (1)-(3), (16) and (17) that $f_\alpha(A) = E(A \otimes \Pi_\alpha)/w_\alpha$ and hence that

$$E(A|\mathcal{M}) = \sum_{\alpha=1}^n E(A \otimes \Pi_\alpha) \Pi_\alpha / w_\alpha.$$

Since Π_α is the projector for the cell \mathcal{K}_α , its coefficient is the conditional expectation value $E(A|\mathcal{K}_\alpha)$ of A , given the macrostate \mathcal{K}_α of \mathcal{I} . Thus

$$E(A|\mathcal{K}_\alpha) = E(A \otimes \Pi_\alpha) / w_\alpha \quad \forall A \in \mathcal{A}, w_\alpha \neq 0. \quad (18)$$

We now relate $E(A)$, $E(A|\mathcal{K}_\alpha)$ and w_α to the key dynamical quantity

$$F_{r,s;\alpha} = \text{Tr}(\Omega_{r,s}(t) \Pi_\alpha) \quad \forall r, s, \alpha \in \{1, \dots, n\}, \quad (19)$$

noting in particular that $F_{r,r;\alpha}$ is the probability that \mathcal{K}_α is the macrostate of \mathcal{I} at time t when u_r is the state of S . By Eqs. (2), (11)-(13), (16), (18) and (19), $E(A)$, $E(A|\mathcal{K}_\alpha)$ and w_α are related to F by the formulae

$$E(A) = \sum_{r=1}^n |c_r|^2 (u_r, A u_r) + \sum_{r \neq s; r, s=1}^n \sum_{\alpha=1}^n F_{r,s;\alpha} \bar{c}_r c_s (u_r, A u_s) \quad \forall A \in \mathcal{A}, \quad (20)$$

$$E(A|\mathcal{K}_\alpha) = \sum_{r,s=1}^n F_{r,s;\alpha} \bar{c}_r c_s (u_r, A u_s) / w_\alpha \quad \forall A \in \mathcal{A}, w_\alpha \neq 0. \quad (21)$$

and

$$w_\alpha = \sum_{r,s=1}^n \bar{c}_r c_s F_{r,s;\alpha}. \quad (22)$$

The following key properties of F ensue from Eqs. (2), (13) and (19).

$$\sum_{\alpha=1}^n F_{r,r;\alpha} = 1, \quad (23)$$

$$1 \geq F_{r,r;\alpha} \geq 0, \quad (24)$$

$$F_{r,s;\alpha} = \overline{F_{s,r;\alpha}} \quad (25)$$

and, for $z_1, \dots, z_n \in \mathbf{C}$, the sesquilinear form $\sum_{r,s=1}^n \bar{z}_r z_s F_{r,s;\alpha}$ is positive. Hence

$$F_{r,r;\alpha} F_{s,s;\alpha} \geq |F_{r,s;\alpha}|^2. \quad (26)$$

3. The Measurement Process

Suppose now that a reading of the pointer of \mathcal{I} is made at time t . Then, according to the standard probabilistic interpretation of quantum mechanics, it follows from the above specifications that

- (i) $E(A)$ is the expectation value of the observable A of S immediately before the reading;
- (ii) w_α is the probability that the reading yields the result that the macrostate of \mathcal{I} corresponds to the cell \mathcal{K}_α ; and,
- (iii) in that case, $E(A|\mathcal{K}_\alpha)$ is the expectation value of A immediately after the measurement.

Hence the conditions for the realisation of the demands (I) and (II) of Von Neumann's phenomenological picture, described in Sec. 1, are that, for t greater than some critical, realistic value, τ , and less, in order of magnitude, than the Poincare recurrence times,

(a)

$$E(A) = \sum_{r=1}^n |c_r|^2 (u_r, Au_r) \quad \forall A \in \mathcal{A} \quad (27)$$

and

(b) there is a unique invertible transformation ϕ of the set $(1, 2, \dots, n)$ such that

$$E(A|\mathcal{K}_\alpha) = (u_{\phi(\alpha)}, Au_{\phi(\alpha)}) \quad \forall A \in \mathcal{A}. \quad (28)$$

In other words, the pointer reading α signifies that the resultant state of S is $u_{\phi(\alpha)}$.

Proposition. *The combination of conditions (a) and (b) is equivalent to the formula*

$$F_{r,r;\phi^{-1}(r)} = 1 \quad \forall r = 1, \dots, n. \quad (29)$$

Hence this formula is equivalent to the Von Neumann conditions (I) and (II).

Proof. By Eqs. (24) and (26), Eq. (29) implies that $F_{r,s;\alpha}$ vanishes unless $r = s = \phi(\alpha)$ and is therefore equivalent to the formula

$$F_{r,s;\alpha} = \delta_{r,\phi(\alpha)} \delta_{s,\phi(\alpha)} \quad \forall r, s, \alpha = 1, \dots, n. \quad (30)$$

Assuming this formula, it follows immediately from Eqs. (20)-(22) that conditions (a) and (b) are fulfilled. Conversely, assuming condition (b), it follows from a comparison of the form of $w_\alpha E(A|\mathcal{K}_\alpha)$ obtained from Eqs. (22) and (28) with that given by Eq. (21) that

$$\sum_{r,s=1}^n \bar{c}_r c_s F_{r,s;\alpha} (u_{\phi(\alpha)}, Au_{\phi(\alpha)}) = \sum_{r,s=1}^n \bar{c}_r c_s F_{r,s;\alpha} (u_r, Au_s) \quad \forall A \in \mathcal{A}, \quad c_1, \dots, c_n \in \mathbf{C}.$$

On equating coefficients of $\bar{c}_r c_s$ in this formula, we see that

$$F_{r,s;\alpha} [(u_{\phi(\alpha)}, Au_{\phi(\alpha)}) - (u_r, Au_s)] = 0 \quad \forall A \in \mathcal{A}, \quad r, s = 1, 2, \dots, n.$$

In view of the orthonormality of the u_r 's and the invertibility of ϕ , this last formula is equivalent to Eq. (30) and thus to Eq. (29).

Weakening of the Condition (29). In fact, that condition is extremely stringent and one sees both from the study of tractable models [2-4] and from a general probabilistic argument, presented in Sec. 4, that it should be weakened by a corrective term, due to macroscopic fluctuations, that decreases exponentially with N for large N . In other words, the sharp condition given by Eq. (29) should be weakened to the form wherein the difference between the two sides of that equation does not exceed $\exp(-cN)$, where c a positive constant of the order of unity. Hence, in view of Eqs. (23) and (24), this weaker condition is that

$$0 \leq 1 - F_{r,r;\phi^{-1}(r)} \leq \exp(-cN). \quad (31)$$

Correspondingly, it follows [2, 3] from arguments parallel to those employed to pass from Eqs. (27) and (28) to Eq. (30) in the proof of the above Proposition that the weakening of condition (29) to Eq. (31) implies corrections of order $\exp(-cN/2)$ to Eqs. (27) and (28), i.e. to the Von Neumann conditions (I) and (II). These corrections are utterly negligible from a physical standpoint, since N is typically of the order of 10^{8d} , where d is the dimensionality of the instrument \mathcal{I} .

A further essential property of an efficacious measuring instrument is that it should be stable against local perturbations of its initial state Ω (cf. [15, 16]). We express this condition in the following form.

S. The formula (31) is stable under perturbations of the initial state Ω of \mathcal{I} that are localised in the sense of leaving this state unchanged outside a ball whose volume is $O(1)$ with respect to N .

Thus, we characterise a quantum measurement process by the conditions given by Eq. (31) and *S*. These conditions are viable, since they have been shown to be fulfilled by concrete models that may be described as follows: we refer the reader to the cited articles for full treatments of them.

Model 1 [2, 3]. This is a finite version of the Coleman-Hepp model [17], which is designed to measure the z -component, say, of the spin of an electron. The model consists of an electron, \mathcal{E} , and a chain, \mathcal{C} , of N Pauli spins. We regard the electron \mathcal{E} as a composite of its own Pauli spin, S , and a spinless particle, \mathcal{P} , that carries its orbital motion. We then take the spin S to be the microsystem under observation and the composite of \mathcal{P} and \mathcal{C} to be the measuring instrument \mathcal{I} . We take the phase cells of \mathcal{I} to be the subspaces \mathcal{K}_{\pm} of its canonically defined Hilbert space \mathcal{K} that correspond to positive and negative polarisations, respectively. We assume that \mathcal{E} and \mathcal{C} are coupled together at $t = 0$ following independent preparations of S and \mathcal{P} in pure states and \mathcal{C} in a state of equilibrium* subject to a constraint, subsequently removed, that fixes its polarisation to a value directed along Oz : the initial state of \mathcal{P} is assumed to be a wave packet localised at one end of \mathcal{C} and moving towards the other end of that chain. The $\mathcal{E} - \mathcal{C}$ coupling, following this preparation, is assumed to be of the form $P_- \otimes V$, where P_- is the projection operator for the eigenstate of S for which the z -component, s_z , of its spin is negative and V is an interaction between

* This is the state that maximises the entropy of \mathcal{C} subject to the specified constraint, and by the subadditivity of entropy [25] it takes the form $\otimes_{n=1}^N \frac{1}{2}(I + m\sigma_{n,z})$, where m is its polarisation and $\sigma_{n,x}$ is the z -component of the n 'th spin of \mathcal{C} .

\mathcal{P} and the spins comprising \mathcal{C} that reverses the latter ones in turn as the electron passes by them. Thus, up to corrections due to fluctuation effects, the resultant macrostate of \mathcal{I} corresponds to the cell \mathcal{K}_+ or \mathcal{K}_- according to whether s_z is in its eigenstate with positive or negative eigenvalue. To be precise [2, 3], taking fluctuations properly into account, the system $\mathcal{I} = (\mathcal{P} + \mathcal{C})$ satisfies the conditions given by Eq. (31) and \mathcal{S} and is therefore an effective measuring instrument for the spin S ; while the time taken to effect the measurement is essentially that required for the electron to run the length of the chain.

Model 2. This is the Allahverdyan-Balian-Nieuwenhuizen (ABN) model [4]. It consists of a localised Pauli spin, S , an Ising-Weiss ferromagnet, \mathcal{F} , and a heat bath \mathcal{B} . As described in the terms of the present article, S is the microsystem under observation and the composite $(\mathcal{F} + \mathcal{B})$ is the instrument \mathcal{I} that measures the z -component, s_z , of its spin. The phase cells of \mathcal{I} are the subspaces \mathcal{K}_\pm of its canonically defined Hilbert space \mathcal{K} that carry positive and negative polarisations, respectively, along Oz . It is assumed that the initial states of S , \mathcal{F} and \mathcal{B} are uncorrelated with \mathcal{F} in a polarisation-free metastable state below its transition temperature and \mathcal{B} in a thermal equilibrium state at that temperature. It is assumed that the subsequent couplings of the components of the model comprise (a) long range interactions between the z -components of the spin of S and those of \mathcal{F} and (b) interactions between \mathcal{B} and \mathcal{F} that drive the latter into an equilibrium state with positive or negative polarisation along Oz according to whether s_z is positive or negative. Under these conditions, \mathcal{I} serves as an effective measuring apparatus for S [4], i.e. it satisfies Eq. (31) and the stability condition*. Again, as in Model 1, the cells \mathcal{K}_\pm represent the pointer positions for the eigenstates of the s_z with positive and negative eigenvalues, respectively.

4. Role of the Large Deviation Principle

We now aim to probe more deeply into the quantum statistics of the measurement process in order to excavate the source of the properties of \mathcal{I} represented by Eq. (31) and the stability condition \mathcal{S} . For simplicity, we confine our attention here to the case where the pointer reading corresponds to the value of a single coarse grained macroscopic observable** M of \mathcal{I} and thus where the algebra \mathcal{M} is generated by bounded functions of M . Further, we assume that M is a coarsened version of a fine-grained extensive observable M_f in the following sense (cf. [22, 23]).

Defining the intensive variable

$$m := N^{-1}M_f, \tag{32}$$

we assume that m is a bounded operator in \mathcal{K} with pure point spectrum, whose highest and lowest points are the extremals of a closed interval Δ of \mathbf{R} . Further, we assume that this spectrum simulates a continuum for large N , in that the maximum spacing between the eigenvalues of m tends to zero as N tends to infinity.

* This condition was not treated in [4], but a simple application of the method of Refs. [2, 3] demonstrates that the ABN model does satisfy it

** Generalisation to the case of several intercommuting ones is straightforward.

We construct the coarse-grained version M of M_f by dividing Δ into a set of disjoint intervals of equal length $\{\mathcal{J}_\alpha | \alpha = 1, \dots, n\}$ and defining \mathcal{K}_α to be the subspace of \mathcal{K} spanned by the eigenvectors of m whose eigenvalues lie in \mathcal{J}_α . We then define m_α to be the arithmetic mean of those eigenvalues and define the coarse-grained observable M by the formula

$$M = \sum_{\alpha=1}^n N m_\alpha \Pi_\alpha, \quad (33)$$

where, as previously, Π_α is the projection operator for \mathcal{K}_α .

We recall that, as noted following Eq. (19), $F_{r,r;\alpha}$ is the probability that \mathcal{K}_α is the macrostate of \mathcal{I} at time t that corresponds to the eigenstate u_r of S . Furthermore, the density matrix $\Omega_{r,r}(t)$, defined by Eq. (13), represents the state of the *full system* \mathcal{I} at time t corresponding to this eigenstate of S . Hence it follows from the above construction of M in terms of the fine-grained intensive observable m , which we may treat as a classical variable, that

$$F_{r,r;\alpha} = P_r(m \in \mathcal{J}_\alpha), \quad (34)$$

where P_r is the probability measure on Δ induced by the state $\Omega_{r,r}(t)$.

We now assume that this probability satisfies a *large deviation principle* [24], which is widely applicable to intensive macroscopic variables of many-particle systems. As noted in Sec. 1, this is a natural generalisation to nonequilibrium situations of Einstein's formula $P = \text{const.} \exp(S)$ for the relationship at thermal equilibrium between the entropy S , expressed as function of macroscopic variables and their probability distribution P . Thus, in the present situation, the large deviation principle asserts that the density of the probability measure P_r on the variable m takes the form $f_{r,N}(m) \exp(N\sigma_r(m))$, where the function σ_r is N -independent and plays the role of a generalised specific entropy and $N^{-1} \log(f_{r,N}(m))$ tends to zero as N tends to infinity. Thus, for large N , the density of P_r is effectively governed by the exponential term $\exp(N\sigma_r)$.

In order to demonstrate that, under very mild conditions on σ_r , both Eq. (31) and the stability condition \mathcal{S} are satisfied, we suppose that

- (a) for each $r \in (1, 2, \dots, n)$, σ_r attains its maximum at precisely one value, m_r , of m ;
- (b) m_r lies in the interior of one of the intervals \mathcal{J}_α , namely $\mathcal{J}_{\phi^{-1}(r)}$, and $\phi^{-1}(r) \neq \phi^{-1}(s)$ if $r \neq s$;
- (c) $\sigma_r(m_r) - \sigma_r(m)$ is greater than some positive constant c for all $r \in (1, 2, \dots, n)$ and $m \notin \mathcal{J}_{\phi^{-1}(r)}$; and
- (d) the function σ_r is unaffected by localised perturbations, as specified in condition \mathcal{S} , of the initial state Ω of \mathcal{I} .

These conditions are certainly viable and, indeed, they are fulfilled by the models of Refs. [2-4]. Further, when taken in conjunction with the above specified large deviation principle for P_r , they imply that a is an invertible transformation of the set $(1, 2, \dots, n)$ and that

$$P_r(m \in \mathcal{J}_\alpha) < \exp(-cN) \text{ for } \alpha \neq \phi^{-1}(r),$$

where c is a positive constant of the order of unity. In view of Eq. (34), this signifies that the instrument \mathcal{I} satisfies the condition (3); and property (d) ensures that it fulfills the stability condition \mathcal{S} .

5. Concluding Remarks

We have provided a general mathematical scheme for the description of quantum measurement theory within the framework of Schroedinger dynamics and quantum probability. This scheme is manifestly realisable and involves nothing more than the dynamics of the composite of a quantum microsystem S and a macroscopic measuring instrument \mathcal{I} . Thus, it does not require any appeal either to cosmological actions or to Von Neumann-Wigner psycho-physical parallelism, with its infinite hierarchy of measuring instruments. Indeed the only participation of human intelligence in the measurement process is in the design of the instrument \mathcal{I} and the interpretation of readings of its pointer positions.

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